Unit 5

Dimensions of Space

Dimension	Looks Like	Description	Units
0^{th}	•	A single point	No unit
1^{st}		A line - length	$Unit^1$
2^{nd}		A flat surface - area	$Units^2$
3^{rd}		A space - volume	$Units^3$

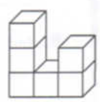
L1 Views of 3D solids

- Spatial sense helps us create, analyze, mentally visualize, classify and transform solids.
- Drawing 3D solids could be challenging but a lot of fun.
- Google:

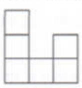
3D street arts.

Views of Cubes

Using cubes, the object on the right is constructed.



Depending on from where you are looking, you can see different views of the object.
The front view of this object is:

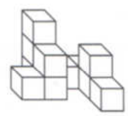
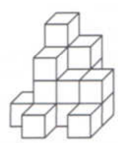
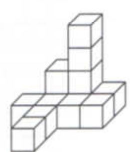


What would the view of this object be

a) from the right? b) from the top? c) from the back?

Views of Cubes

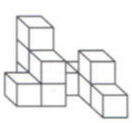
1. Draw the requested views for each of the following solids.

	Front	Right	Top
a) 			
b) 			
c) 			

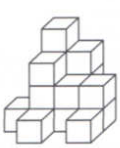
Coded blueprint of a solid:

- (only for the top/bottom view)
- It indicates in each square of the base of the solid, the number of cubes stacked up vertically over it .

a)



b)

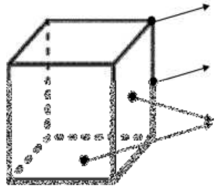


Practice:
5.2 # 1-4

Unit 5

L2 Properties of 3D

Cube



Vertex/Vertices (8)

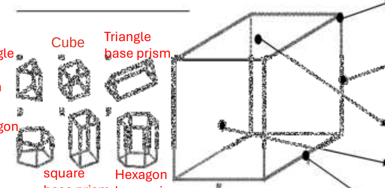
Edge (12)

Faces (6): Lateral (4)

Bases (2)

Prism

Square base prism



Named after the shape of the base

Vertex/Vertices (8)

Edge (12)

Lateral faces (4)

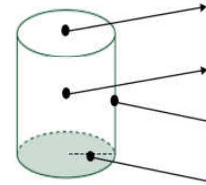
Bases (only 2)

Height (distance between bases)

Cylinder

No Vertices

No Edges



Bases (2)

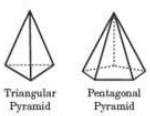
Lateral surface

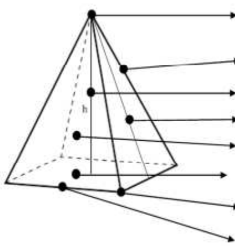
Height (distance between bases)

Radius

Pyramid

Named after the shape of the Base





Apex (1)

Lateral edge (4)

Height *

Slant Height *

Lateral face (4)

Base (only 1)

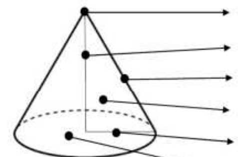
Vertex (Vertices (4)

Base edge/length

$$* \, s^2 = \sqrt{(h^2 + \left(\frac{b}{2}\right)^2)}$$

Cone

No Edges



Apex (1)

Height *

Slant Height *

Lateral surface

Radius

Base (only 1)

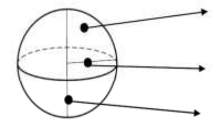
$$* \, s^2 = \sqrt{(h^2 + r^2)}$$

Sphere

No Vertices

No Edges

No Bases

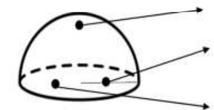


Lateral surface

Radius

Diameter

Hemisphere

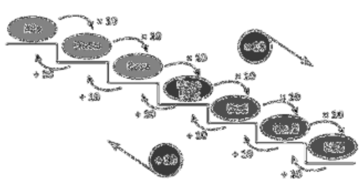


Lateral surface

Radius

Base (maybe)

Unit conversion



- Always check that units are matching before you start a question
- To convert Area (unit^2) multiply or divide by 100 (10^2)
- To convert Volume (unit^3) multiply or divide by 1000 (10^3)

A) **Volume** is the quantity of space that an object occupies

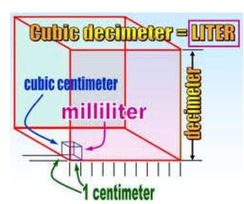
Capacity is the volume of the hollow part of an object.

B) The cubic decimetre (dm^3) corresponds to a cube with edge 1 dm (10 cm)

1 m^3 = 1 KL

1 dm^3 = 1 L

1 cm^3 = 1 mL



C) 1 L of water = 1 kg

at standard temperature and pressure

Practice:
5.4 # 1,2

Unit 5

L3 Areas of Prisms and Cylinders

For every solid we can find two types of areas:

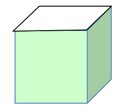
- Lateral area (A_L): area of lateral faces-no base(s)
- Total area (A_T): area of lateral faces and base(s)

Formulas:

	A_L (Lateral Area)	$A_T = A_L + 2A_b$ (Total Area)
Cubes		
Prisms		
Cylinders		

	A_L (Lateral Area)	$A_T = A_L + 2A_b$ (Area total)
Cubes	$4w^2$	$6w^2$

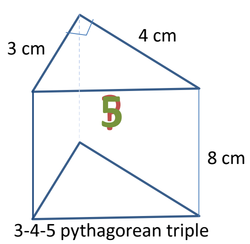
Ex 1: Find the total area



3 cm

	A_L (Lateral Area)	$A_T = A_L + 2A_b$ (Area total)
Prisms	$P_b h$	$P_b h + 2A_b$

Ex 2: Find the total area:



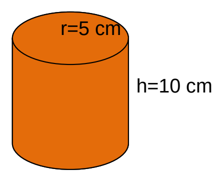
$A_T = P_b h + 2A_b$

$P_b = 3+4+5 = 12$

$A_b = (3)(4)/2 = 6$

	A_L (Lateral Area)	$A_T = A_L + 2A_b$ (Area total)
Cylinders	$2\pi rh$ or πdh	$2\pi rh + 2\pi r^2$

Ex 3: Find the total surface area of the cylinder

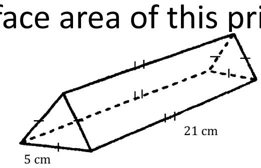


$A_T = 2\pi rh + 2\pi r^2$

	A_L (Lateral Area)	$A_T = A_L + 2A_b$ (Area total)
Prisms	$P_b h$	$P_b h + 2A_b$

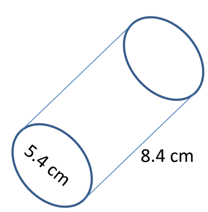
Ex 4: Find the total surface area of this prism

$A_T = P_b h + 2A_b$



	A_L (Lateral Area)	$A_T = A_L + 2A_b$ (Area total)
Cylinders	$2\pi rh$ or πdh	$2\pi rh + 2\pi r^2$

Ex 5: Find the total area:

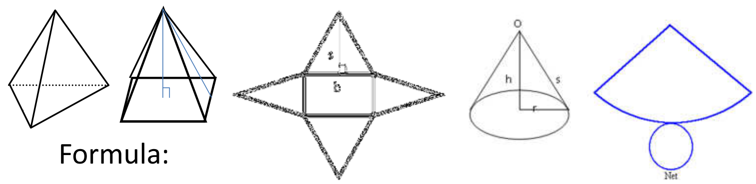


If d = 5.4 cm
Then r = 2.7 cm

Practice: 5.5 Set 1

Unit 5

L4 Areas of Pyramids and Cones

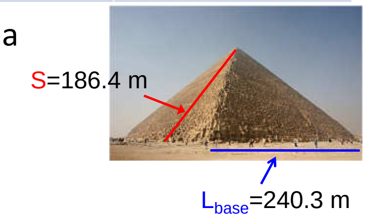


Formula:

	A_L (Lateral Area)	$A_T = A_L + A_b$ (Area total)
Pyramid	$\frac{P_b s}{2}$	$\frac{P_b s}{2} + A_b$
Cone	$\pi r s$	$\pi r s + \pi r^2$

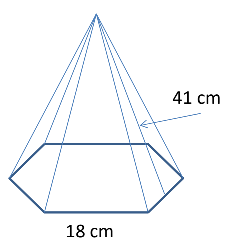
	A_L (Lateral Area)	$A_T = A_L + A_b$ (Area total)
Pyramid	$\frac{P_b s}{2}$	$\frac{P_b s}{2} + A_b$

Ex 1: Find the surface area of the pyramid of Giza.



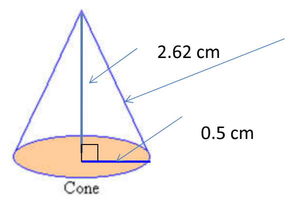
	A_L (Lateral Area)	$A_T = A_L + A_b$ (Area total)
Pyramid	$\frac{P_b s}{2}$	$\frac{P_b s}{2} + A_b$

Ex 2: Calculate the Lateral Area



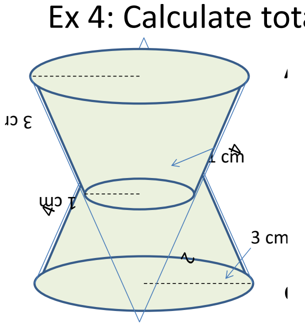
	A_L (Lateral Area)	$A_T = A_L + A_b$ (Area total)
Cone	$\pi r s$	$\pi r s + \pi r^2$

Ex 3: Find the total surface area of the cone.



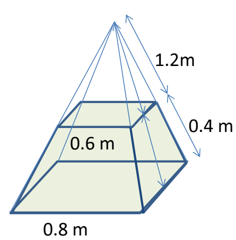
	A_L (Lateral Area)	$A_T = A_L + A_b$ (Area total)
Cone	$\pi r s$	$\pi r s + \pi r^2$

Ex 4: Calculate total area both bases included



Ex 5: A factory makes cement blocks in the shape of truncated square base pyramids. They want to paint the lateral faces and the upper base of these blocks. How many cans of paint will be required if each can on paint covers 40 m^2 . They will make 1000 blocks?

$$A_T = A_{L\text{ BIG PYR}} - A_{L\text{ Small pyr}} + A_{\text{upper base}}$$



Practice: 5.5 - Set 2

Unit 5

L5 Area of Spheres

- Formulas:

	A_L (Lateral Area)	$A_T = A_L + A_b$ (Only hemispheres have a base)
Spheres	$4\pi r^2$	
Hemispheres	$2\pi r^2$	$3\pi r^2$

	$A_L = A_T$ Sphere (Lateral Area)	$A_T = A_L + A_b$ (only hemispheres have a base)
	$4\pi r^2$	$2\pi r^2 + \pi r^2$

Ex 1: A Tennis ball has diameter 6 cm. What is its surface area?

6 cm

	$A_L = A_T$ Sphere (Lateral Area)	$A_T = A_L + A_b$ (only hemispheres have a base)
	$4\pi r^2$	$2\pi r^2 + \pi r^2$

Ex 2: A soccer ball with diameter 30 cm is placed tightly inside a cube box. Find the difference between their surface areas.

30 cm

	$A_L = A_T$ Sphere (Lateral Area)	$A_T = A_L + A_b$ (only hemispheres have a base)
	$4\pi r^2$	$2\pi r^2 + \pi r^2$

Ex 3: How many times greater is the SA of the Sun than the Earth?

d_S=1 391 000 km

r = 695500

d_E=12 756 km r = 6378

Practice:

5.5 Set 3

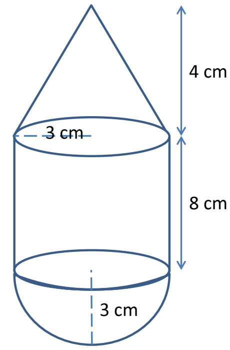
Unit 5

L6 Areas of Composite solids

Unfamiliar Solids can be broken down into simpler solids so that their area and volume can be calculated more easily.




Ex 1:



Note the hidden bases between the cone, cylinder, and the hemisphere.

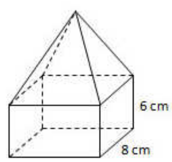
When you have a composite solid:

1. Separate and identify the solids involved.
2. Write the formulas for all the lateral areas involved, and only the visible bases.
3. Calculate them and add them together.
4. Watch out for hidden bases whose areas should not be included.


Ex 2:

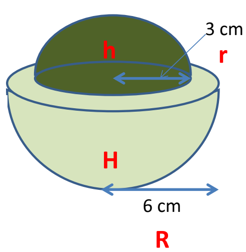
The solid shown below consists of a square base pyramid joined to a square base prism. The height of each lateral surface of the pyramid is 12 cm, and the prism's dimensions are 8 cm by 8 cm and a height of 6 cm. Find the total surface area.

Note the hidden base between the pyramid and the prism.



Ex 3:

A hemisphere is placed on the flat surface of another hemisphere. What is, rounded to the nearest unit, the total area of this solid?



Practice:
5.5 Set 4

Unit 5

L7 Volumes of Prisms & Cylinders

Consider a pack of printer papers. With dimensions:

l = 28 cm; w = 21.5 cm; h = 5 cm.

Area of one paper: A = lw = (28)(21.5) = 602 cm^2

The space this pack occupies (volume):

= (602)(5) = 3010 cm^3

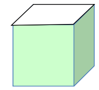
$V_{prism} = A_b \bullet h$

$V_{cube} = w^3$

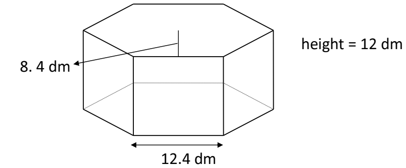
$V_{cylinder} = \pi r^2 \bullet h$

Ex 1: Determine the volume of this cube

$V_{cube} = w^3$



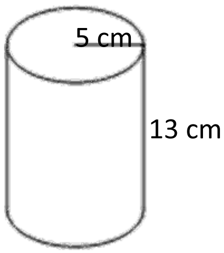
Ex 2: Determine the volume of this prism



$V_{prism} = A_b \bullet h$

Ex 3: Determine the volume of this cylinder

$V = \pi r^2 \bullet h$



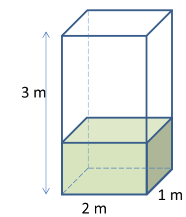
Ex 4: Find the volume of this hot air balloon

D = 14 m
R = 7 m
H = 35 m

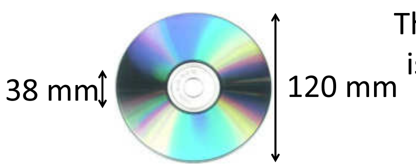
Diameter = 14 m
Height is 2.5 times the diameter

Ex 5:
This tank is filled with 1500 L of gas. How much time will be required to fill the rest of it at a rate of 20 L/min?

$V_{tank} = A_b \bullet h$



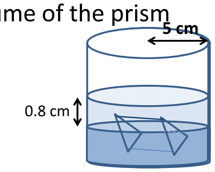
Ex 6: Determine the volume of material required to make a cd in **mL**



Thickness is 4 mm

r = 19 mm
R = 60 mm
h = 4 mm

Ex 7:
A triangular base prism is submerged in a cylindrical bucket of water with a 5 cm radius. The water level in the bucket rises 0.8 cm. What is the volume of the prism submerged in the water?



Practice:
5.6 Set 1

Unit 5

L8 Volumes of Pyramids and Cones

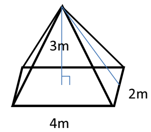
How does the **volume** of any **pyramid** (or **cone**) compare to the volume of a prism (or cylinder) of the same size of base and height?

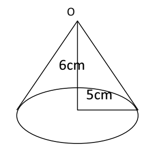
$$V = \frac{A_b \bullet h}{3}$$

$$V_{pyramid} = \frac{A_b\, h}{3} \qquad V_{cone} = \frac{\pi r^2 h}{3}$$

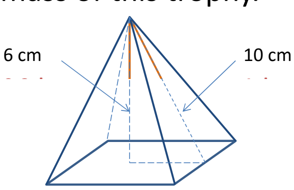
$$V_{pyramid} = \frac{A_b\, h}{3} \qquad V_{cone} = \frac{\pi r^2 h}{3}$$

Ex 1: Find the volume



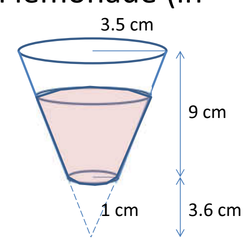


Ex 2:
A square base pyramid trophy, has height 6 cm and its slant height is 10 cm. If it is made of aluminum and the mass of 1 dm^3 of aluminum is 2.7 Kg, calculate the mass of this trophy.



$$V_{pyramid} = \frac{A_b\, h}{3}$$

Ex 3:
If Eric fills the plastic cup (shown below) to $\frac{3}{4}$ its capacity with lemonade, how much lemonade (in cl) will be poured into the cup?



Practice:
5.6 Set 2

Unit 5

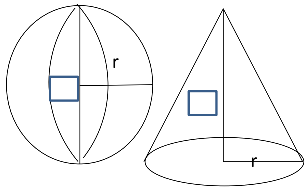
L9 Volume of a Sphere

$V_{sphere} = 2\ V_{cone}$

$= 2(\frac{\pi r^2 \bullet h}{3})$

$= 2(\frac{\pi r^2 \bullet 2r}{3})$

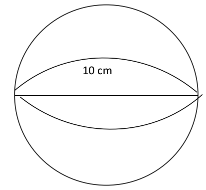
$= \frac{4\pi r^3}{3}$



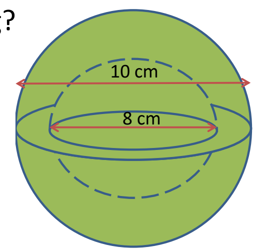
$\boxed{V_{sphere} = \frac{4\pi r^3}{3}}$

$V_{sphere} = \frac{4\pi r^3}{3}$

Ex 1: Find the volume of the sphere with diameter 10 cm



Ex 3:
The interior and exterior diameters of a metallic ball are 8 cm and 10 cm respectively. What is the ball's mass if the mass of 1 dm^3 of this metal is 3 Kg?



Ex 2: Find the volume of the Earth in km^3

diameter= 12 756 km

r = 6 378 km

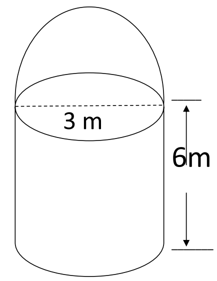
Practice:
5.6 Set 3

Unit 5

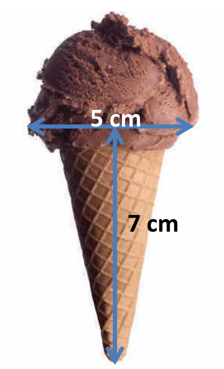
L10 Volumes of composite solids

1. Separate and identify the solids involved.
2. Write the formulas for all the solids involved.
3. Calculate them and add them together.

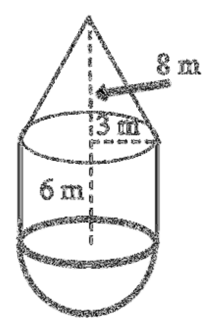
Ex 1: Find the volume of this silo in Kl:



Ex 2: Find the volume of this ice cream cone in ml:



Ex 3: Determine the volume of this space probe



Practice:
5.6 Set 4

SOLIDS	LATERAL AREA	TOTAL AREA	Volume
RIGHT PRISMS	$A_{LAT} = P_B \bullet h$	$A_{TOT} = P_B\, h + 2A_B$	$V_{prism} = A_b \bullet h$
RIGHT CYLINDERS	$A_{LAT} = 2\pi rh$	$A_{TOT} = 2\pi rh + 2\pi r^2$	$V_{cylinder} = \pi r^2 \bullet h$
RIGHT REGULAR PYRAMIDS	$A_{LAT} = \frac{P_b\, s}{2}$	$A_{TOT} = \frac{P_b\, s}{2} + A_b$	$V_{pyramid} = \frac{A_b \bullet h}{3}$
RIGHT CONES	$A_{LAT} = \pi rs$	$A_{TOT} = \pi rs + \pi r^2$	$V_{cone} = \frac{\pi r^2 \bullet h}{3}$
SPHERES	$A_{LAT} = A_{TOT} = 4\pi r^2$		$V_{sphere} = \frac{4\pi r^3}{3}$
HEMISPHERE	$A_{LAT} = A_{TOT} = 2\pi r^2$ Note: if the base is included, add πr^2		$V_{sphere} = \frac{2\pi r^3}{3}$